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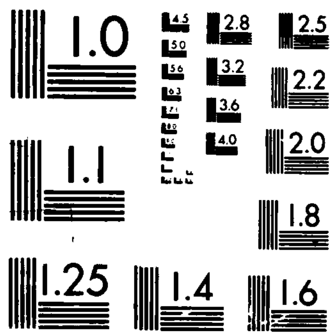
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Scaling Laws for Geomagnetic Tail Current Sheet Acceleration

CHRISTOPHER SHERMAN

22 July 1987

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PROJECT 2310

AIR FORCE GEOPHYSICS LABORATORY

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Solutions of the equation describing proton acceleration in the electric field of the Earth's geomagnetic tail often require lengthy numerical calculations and parametric exploration can be difficult. Here, we present considerations of a scaling nature and show that these considerations may be used to extend, correlate, organize, and verify groups of solutions with regard to parametric variation.

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Preface

My thanks to John Retterer for several most helpful suggestions.

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1. INTRODUCTION

Lyons and Speiser[1] have calculated proton accelerations in the electric cross field of the geomagnetic tail. Although these calculations are fairly extensive, as the authors comment, there is still a fair amount of unexplored territory as far as combinations of relevant parameters are concerned. The purpose of this report is to present considerations of a scaling nature and to show that these considerations may be profitably utilized to correlate, organize, and verify groups of solutions with regard to parametric variation.

2. DERIVATION AND DISCUSSION OF GENERAL SCALING LAWS

Although the equations involved in these calculations are available, it is not necessary to refer to them to obtain scaling/dimensional information. This latter can be accomplished by use of the Buckingham pi theorem, which is stated here without proof. Details may be found in Langhaar,[2] pp. 18 and 47 ff. The theorem is:

(Received for Publication 16 July 1987)

1. Lyons, L.R., and Speiser, T.W. (1982) Evidence for current sheet acceleration in the geomagnetic tail, J. Geophys. Res. 87:2276.
2. Langhaar, Henry L. (1951) Dimensional Analysis and Theory of Models, Wiley, New York.

If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products. If the number of input parameters is n, and the number of fundamental dimensions is r, then the number of dimensionless products in the set will be $p = n - r$.

There are eight parameters involved in the solutions of the Lyons and Speiser equations, B_z, B_{xL}, E, e, m, $V_\parallel$, $V_\perp$, d. Of these, the first five are equation parameters, the next two initial conditions, and the last, both. These parameters are:[1]

B_z	Constant magnetic field normal to tail current sheet.
$\lvert B_{xL} \rvert$	Constant magnetic field in x direction for $\lvert z \rvert \geq d$.
E	Constant electric field normal to B_z and B_{xL}.
e	Particle charge.
m	Particle mass.
$V_\parallel$	Initial component of velocity along the magnetic field.
$V_\perp$	Initial component of velocity normal to the magnetic field.
d	Sheet half-thickness in z direction.

Since the number of fundamental units is four (length, mass, time, and charge), there are at most 8 - 4 = 4 independent dimensionless parameters that may appear in the solutions for any unknown quantity. These may be formed in a multiplicity of ways, and we choose, for our purposes, the following:

$$B_{xL}/B_z;\ V_\perp/V_\parallel;\ (V_\parallel^2 + V_\perp^2)^{1/2}\ B_z/E;\ m/e\ E/B_z^2 d$$

The meaning of the first two is clear, and the third is a ratio of initial velocities to a drift velocity. The last, by multiplication above and below by E, is seen to be a ratio of kinetic energy of drift, to the drop in potential energy, Ed, across the half-sheet.

We intend to concentrate our attention on results shown in Figure 8 of Lyons and Speiser,[1] and this figure is reproduced and enlarged in Figure 1. Table 1 lists the parameters characterizing the curves shown in the figure. Using the above dimensionless groups, we form an expression for U_I, the energy increase that is plotted as ordinate,

$$U_I = eEd\ F[(B_{xL}/B_z),\ (V_\perp/V_\parallel),\ (V_\parallel^2 + V_\perp^2)^{1/2}\ B_z/E),\ (m/e\ E/B_z^2 d)] \qquad (1)$$

Here, F is an unspecified function, and, hence, the quantity preceding it may be chosen arbitrarily, subject only to the restriction that it have the dimensions of energy.

This general expression for the energy increase allows one to present all available information on U_I in terms of the four dimensionless parameters chosen. It is possible, and Lyons and Speiser[1] indicate, that U_I will not depend on <u>all</u> of

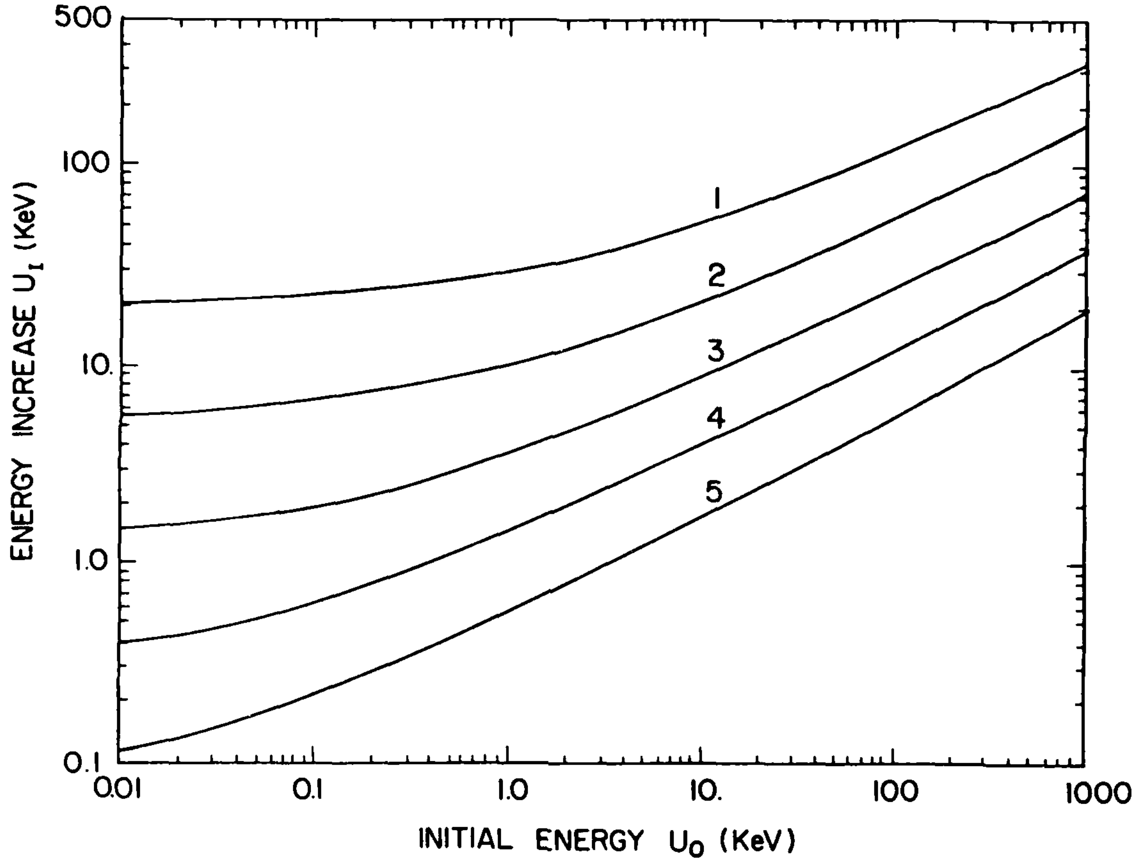

Figure 1. Energy Increase U_I Versus Initial Energy U_O, for Five Sets of Input Parameters (Redrawn From Lyons and Speiser, 1982)

these parameters; that is, the functional dependence may be a constant for some of them. As we shall see, however, such conclusions may imply further restrictions on form; and these must all constitute a picture self-consistent and compatible with actual solutions. From Eq. (1), certain scaling laws are evident. Thus, for example, if $V_{\parallel}$, $V_{\perp}$ are unchanged; E, B_z, B_{xL}, are multiplied by a; and d by 1/a, then U_I remains unchanged. Such a scaling law forms a means of checking the consistency of solutions. Unfortunately, Lyons and Speiser used only two values for d, and these did not have the proper ratio to test the law. A second law is the following: If E is multiplied by a^2; $\mathbf{B}_z$, B_{xL}, $V_{\perp}$, $V_{\parallel}$ are multiplied by a; and d is unchanged, then U_I is multiplied by a^2. Again, there is not, in the present instance, the proper combination of parameters available to test this. If would be of interest in future calculations to do so.

Table 1. Parameters Characterizing the Curves Shown in Figure 1

Curve #	E (v/m)	B_z (γ)	E/B_z
1	1×10^{-3} 2.5×10^{-4}	1.0 0.25	1×10^{-3}
2	5×10^{-4} 2.5×10^{-4}	1.0 0.5	5×10^{-4}
3	2.5×10^{-4}	1.0	2.5×10^{-4}
4	1.25×10^{-4} 2.5×10^{-4}	1.0 2.0	1.25×10^{-4}
5	6.25×10^{-5} 2.5×10^{-4}	1.0 4.0	6.25×10^{-5}
$B_{XL} = 20\gamma$: d = 1000 km: α eject = 50			

3. APPLICATION TO DATA OF LYONS AND SPEISER

Lyons and Speiser[1] present some evidence to indicate that U_I: (1) is independent of B_{xL}, and (2) depends not on E and B_z individually, but only the ratio E/B_z. This can only be possible if the functional dependence on the group $(m/e\ E/B_z^2 d)$ is linear, in which case, with B_{xL}/B_z a constant, Eq. (1) becomes

$$U_I = m\,(E/B_z)^2\,F\,[(V_{\parallel}{}^2 + V_{\perp}{}^2)^{1/2}\,B_z/E,\ (V_{\perp}/V_{\parallel})] \tag{2}$$

Thus, the above two constraints lead automatically to the further result, that U_I is independent of d. This result is confirmed by their calculations, which show that U_I is close to invariant for a 5:1 change in d.

The second general scaling law, when applied to Eq. (2), becomes: If E is multiplied by a^2; B_z, $V_{\perp}$, $V_{\parallel}$ multiplied by a; then U_I is multiplied by a^2. Here, data is available to check the law, and the results of such a comparison are shown in Figure 2.

The set of points marked "x" scales the two curves marked 1 and 2 in Figure 1, for a value of "a" = 2. A value (plotted as abscissa on Figure 2) is chosen for U_o

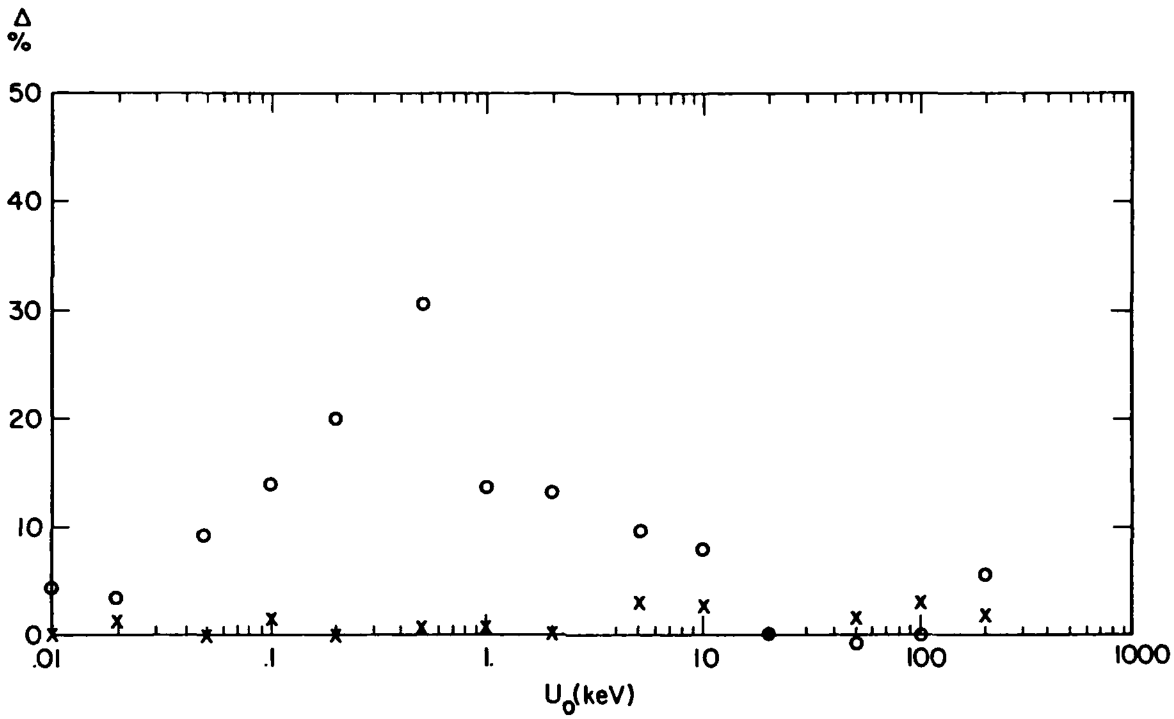

Figure 2. Deviation From Scaling Law, Δ Versus Initial Energy U_0 (Data From Figure 1)

and a corresponding value $U_I)_2$ (second subscript refers to curve number) is read from the graph of Figure 1. Then, for an initial energy $4U_o$, the corresponding value $U_I)_1$ is read from the figure. Δ is then defined as

$$\Delta = \frac{U_I)_1 - 4U_I)_2}{U_I)_1}\ 100\% \qquad (3)$$

If Δ = 0, the law is confirmed for a = 2. It is seen that these two curves scale very closely.

For the set of points marked "o", the same procedure is followed, using the lower two curves, #4 and #5. In this case, there is a systematic departure from the scaling law, amounting to ~ 30 percent at the maximum, for an initial energy U_o = 0.5 keV.

Although allowances must be made for the inaccuracy of reading values from a graph (the line thickness amounts to some 7 percent of the ordinate value), the consistently low values of Δ for the curves 1 and 2 and the higher ones for curves 4 and 5 indicate that a genuine discrepancy exists here. Further, the fact that for curves 1 and 2, and 14 values of U_o, in every case, Δ ≥ o cannot easily be ex-

plained on a statistical basis. Thus, it is possible that, even for curves 1 and 2, there is a slight deviation from proper scaling.

To the extent that this scaling law is obeyed, it shows that these curves are not independent, but can be generated one from the other; and it also provides a consistency check on the calculations. To the extent that the law is violated, it shows a lack of consistency in the assumptions on parameter dependence and/or possible errors in the calculations themselves. Formulas such as Eq. (2) also offer a simple means of constructing empirical formulas to represent the results of numerical calculations.

Martin[3] has made similar calculations, but with a variation in the x as well as the z component of the magnetic field. The x-component of the field is still characterized by a single parameter, so that the total number of equation parameters appearing in a dimensional analysis remains unchanged. However, another initial coordinate, in the x direction, is now needed to completely specify the initial value problem, and this will result, in general, in an additional dimensionless group. These calculations, contrary to those of Lyons and Speiser,[1] show substantial variations in U_I with plasma sheet thickness. These calculations are all for a fixed initial $x = x_o$, and hence, the above derived scaling laws are still valid. We therefore conclude that U_I will depend not only on the ratio E/B_o (B_o is, here, a constant characterizing the strength of the magnetic field), but must also depend on E and/or B_o independently. There is not, however, sufficient data to test this law. Here too, a knowledge of scaling laws would have been advantageous in correlating, checking internal consistency, and minimizing the number of calculations required for parametric exploration.

These laws offer a useful adjunct to such calculations of this type as may be undertaken in the future.

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3. Martin, R.J., Jr. (1986) The effect of plasma sheet thickness on ion acceleration near a magnetic neutral line, in Ion Acceleration in the Magnetosphere and Ionosphere, American Geophysical Union, Washington, D.C., pp. 141-145.

References

1. Lyons, L.R., and Speiser, T.W. (1982) Evidence for current sheet acceleration in the geomagnetic tail, J. Geophys. Res. 87:2276.

2. Langhaar, Henry L. (1951) Dimensional Analysis and Theory of Models, Wiley, New York.

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